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HEAT TRANSFER AND LATERAL PARTICLE MIGRATION IN SUSPENSION FLOW THROUGH A CONVERGENT CHANNEL

S. H. EMERMAN
Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, NY 14853, U.S.A.

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Abstract—The migration of particles across fluid streamlines has been observed to occur in a variety of suspension flows. This lateral particle migration can either heat or cool the suspension depending upon the differences in the specific heat capacities of the fluid and solid phases, and the direction of migration with respect to the lateral temperature gradient. Rigid spheres and rodlike particles migrate toward the wall in the creeping flow of a Newtonian fluid through a convergent channel. The temperature drop across a convergent channel is calculated and shown to be negligibly small for the injection molding of a polymer composite.

1. INTRODUCTION

Fluids containing solid particles occur throughout the physical and biological sciences. Under certain circumstances the particles migrate across the fluid streamlines. Gauthier *et al.* (1971) have observed lateral particle migration in Couette and Poiseuille flows of power-law and elasticoviscous fluids. In power-law fluids, particles migrate towards the region of higher velocity gradient while in elasticoviscous fluids, the particles migrate toward the region of lower velocity gradient. In the pipe flow of a Newtonian fluid at moderate Reynolds number, Segré & Silerberg (1961) observed the accumulation of particles at some radius between the wall and the axis. Faxen's law and its recent extensions make it possible to calculate particle migration in a creeping Newtonian flow containing rigid spheres or prolate spheroids.

The purpose of this paper is to calculate the particle migration in a convergent channel in order to study the effect of particle migration upon heat transfer. We will not consider wall effects, buoyancy effects or interactions between particles. The second section of this paper will derive a differential equation relating particle migration rate to heat transfer. The third section will calculate the migration of rigid spheres or prolate spheroids in the creeping flow of a Newtonian fluid through a convergent channel. The fourth section will compute the temperature change across a convergent channel due to particle migration, and the fifth section will apply the results to the injection molding of polymer composites.

2. HEAT TRANSFER

Consider the steady-state one-dimensional flow of a fluid containing volume concentration ϕ of solid particles (see figure 1). The x- and y-coordinates are parallel and perpendicular (respectively) to the rectilinear streamlines. The ambient fluid flow has velocity $\mathbf{v} = v\hat{x}$; the particles have velocity $\mathbf{U} = U_x\hat{x} + U_y\hat{y}$. We assume that the particle velocity in the x-direction, U_x, differs only slightly from the ambient fluid velocity v. The particles migrate across the fluid streamlines with velocity U_y. In order to conserve mass, there must be a return flow of fluid at velocity $\phi U_y/(1-\phi)$, perpendicular to the bulk flow direction and opposite to the direction of particle migration. Both fluid and particles have density ρ. The solid particles have specific heat capacity c_s, the fluid has specific heat capacity c_f and the suspension has an effective heat capacity c_e given by

$$c_e = c_f(1-\phi) + c_s\phi. \quad [1]$$

We assume the fluid and particles to be in thermal equilibrium at temperature T.

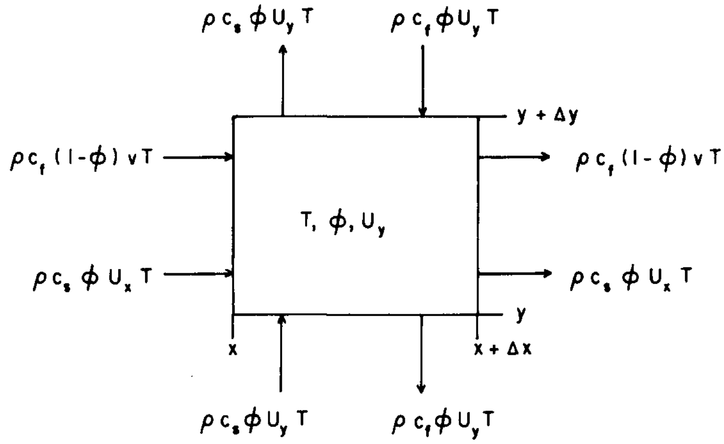

Figure 1. Energy balance in the control volume.

Figure 1 illustrates the energy balance in a control volume fixed in space. Since we wish to study only the effects of particle migration, we have not included thermal conduction or shear heating. Assuming no net flux of energy into the control volume, we obtain by inspection of figure 1:

$$(c_f - c_s)\frac{\partial}{\partial y}(\phi U_y T) = c_f \frac{\partial}{\partial x}[(1-\phi)vT] + c_s \frac{\partial}{\partial x}(\phi U_x T). \qquad [2]$$

Conservation of fluid and particles requires that

$$\frac{\partial}{\partial x}[v(1-\phi)] - \frac{\partial}{\partial y}(\phi U_y) = 0 \qquad [3]$$

and

$$\frac{\partial}{\partial x}(\phi U_x) + \frac{\partial}{\partial y}(\phi U_y) = 0. \qquad [4]$$

Combining [1], [3] and [4] with [2] and setting $U_x = v + \epsilon$ yields

$$(c_e v + c_s \phi \epsilon)\frac{\partial T}{\partial x} = (c_f - c_s)\phi U_y \frac{\partial T}{\partial y}. \qquad [5]$$

Since $\epsilon \ll v$, we can neglect the second term on the l.h.s. of [5] to obtain

$$v\frac{\partial T}{\partial x} = \left(\frac{c_f - c_s}{c_e}\right)\phi U_y \frac{\partial T}{\partial y}. \qquad [6]$$

Equation [6] can be rewritten as

$$\frac{dT}{dt} = \left(\frac{c_f - c_s}{c_e}\right)\phi U_y \frac{\partial T}{\partial y}, \qquad [7]$$

where the operator "d/dt" represents the total derivative of a material point of fluid. Equation [7] gives the rate of change in temperature of a material point of fluid as it travels along a streamline through spatially varying U_y, ϕ and $\partial T/\partial y$. Terms representing thermal conduction or shear heating can readily be added to the r.h.s. of [7]. Note that [7] has been derived only for a steady state in which all variables are independent of time at a fixed point.

It will be shown in the next section that, for convergent flow, particles migrate toward the wall. In injection molding the temperature normally decreases toward the wall; thus, the product $U_y(\partial T/\partial y)$ is negative. For most polymer composites, $c_f > c_s$. Therefore, the effect of particle migration is to cool a polymer composite as it flows from the large to the small end of a convergent channel.

3. PARTICLE MIGRATION

In order to calculate particle migration we first solve for the two-dimensional creeping flow between plates which intersect at an angle α (see figure 2). Conservation of momentum and mass in cylindrical coordinates require

$$\frac{\partial p}{\partial r} = \mu\left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{1}{r}\frac{\partial v_r}{\partial r} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} - \frac{v_r}{r^2}\right), \quad [8]$$

$$\frac{1}{r}\frac{\partial p}{\partial \theta} = \mu\left(\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{1}{r}\frac{\partial v_\theta}{\partial r} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2}\right) \quad [9]$$

and

$$\frac{\partial v_r}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} = 0, \quad [10]$$

where r is the distance from the apex in the plane of flow, θ is the angle between the r-axis and the plane bisecting the channel, v_r is the velocity in the r-direction, v_θ is the velocity in the θ-direction, p is the pressure and μ is the dynamic viscosity. The no-slip boundary conditions are

$$v_r = v_\theta = 0 \quad \text{at} \quad \theta = \pm\frac{\alpha}{2}. \quad [11]$$

The solution of [8]–[11] is

$$\left.\begin{aligned} v_r &= \frac{A}{r}[\cos(2\theta) - \cos(\alpha)] \\ v_\theta &= 0, \end{aligned}\right\} \quad [12]$$

where A is a constant proportional to the mass flux. Positive A indicates divergent flow while negative A indicates convergent flow. Since the streamlines are lines of constant θ, lateral particle migration is given by particle velocity in the θ-direction.

Faxen (1924) showed the translational and angular velocity of a rigid sphere suspended in an unbounded creeping flow to be

$$\mathbf{U} = \mathbf{v}_o + \frac{R^2}{6}\nabla^2\mathbf{v}_o \quad [13]$$

and

$$\boldsymbol{\omega} = \frac{1}{2}\nabla \times \mathbf{v}_o, \quad [14]$$

where $\mathbf{U}$ is the translational velocity, $\boldsymbol{\omega}$ is the angular velocity, R is the sphere radius, $\mathbf{v}$ is the ambient velocity field and the subscript "o" indicates evaluation at the center of the sphere. The quantity $\mathbf{v}_o$ is the fluid velocity at the position of the center of the sphere which would occur if the sphere were not present. It is clear from [13] that lateral migration does not occur in flow

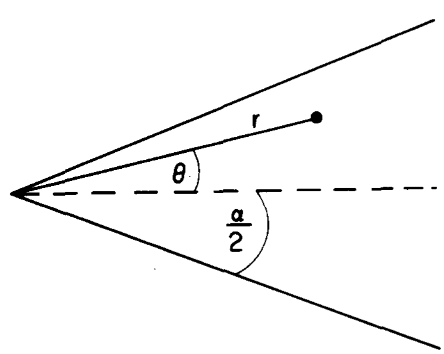

Figure 2. Geometry for two-dimensional flow between plates intersecting at an angle α.

between parallel walls. Inserting [13] into [12] we find the lateral migration rate for convergent flow:

$$U_\theta = \frac{-2AR^2}{3r^3}\sin(2\theta). \tag{15}$$

Spheres migrate toward the walls in convergent flow and toward the axis in divergent flow.

Brenner (1964, 1966) obtained $\mathbf{U}$ and $\boldsymbol{\omega}$ for an ellipsoid in terms of an infinite series of differential operators. Such a formulation is convenient for calculations in Couette or Poiseuille flow since the series truncates after several terms (Brenner & Haber 1983). However, for a nonpolynomial flow such as [12] the infinite series is difficult to evaluate. Recently, Hasimoto (1983) and Kim (1985) independently derived an integral formula for the translational and angular velocity of a rigid prolate spheroid suspended in an unbounded creeping flow:

$$\mathbf{U} = \frac{1}{2c}\int_{-c}^{c}\left[1 + (c^2 - \xi^2)\frac{(1-e^2)}{4e^2}\nabla^2\right]\mathbf{v}(\xi)\,\mathrm{d}\xi \tag{16}$$

and

$$\boldsymbol{\omega} = \frac{3}{8c^3}\int_{-c}^{c}(c^2 - \xi^2)\nabla\times\mathbf{v}(\xi)\,\mathrm{d}\xi + \frac{3}{4c^3}\frac{e^2}{(2-e^2)}\int_{-c}^{c}(c^2-\xi^2) \times\left[1 + (c^2-\xi^2)\frac{(1-e^2)}{8e^2}\nabla^2\right]\mathbf{d}\times\mathbb{e}(\xi)\cdot\mathbf{d}\,\mathrm{d}\xi, \tag{17}$$

where ξ is the distance along the spheroid axis ($\xi = 0$ indicates the spheroid center), c is the distance from center to focus, $e = c/a$ is the eccentricity (a is the length of the major semiaxis), $\mathbf{d}$ is the unit vector denoting the orientation of the spheroid axis and $\mathbb{e}$ is the rate-of-strain tensor. Equations [16] and [17] are the Faxen laws for force-free and torque-free spheroids. It is clear from [16] that lateral migration does not occur for flow between parallel walls.

For most applications a number of simplifications are possible. Let $b = (a^2 - c^2)^{1/2}$ be the length of the minor semiaxis and l the length scale of flow. For example, for a particle in a convergent flow the length scale is r. If $(b/l)^2 \ll 1$, then the terms involving the Laplacian operator in [16] and [17] can be dropped, yielding

$$\mathbf{U} = \frac{1}{2c}\int_{-c}^{c}\mathbf{v}(\xi)\,\mathrm{d}\xi \tag{18}$$

and

$$\boldsymbol{\omega} = \frac{3}{8c^3}\int_{-c}^{c}(c^2-\xi^2)\nabla\times\mathbf{v}(\xi)\,\mathrm{d}\xi + \frac{3}{4c^3}\frac{e^2}{(2-e^2)}\int_{-c}^{c}(c^2-\xi^2)\mathbf{d}\times\mathbb{e}(\xi)\cdot\mathbf{d}\,\mathrm{d}\xi. \tag{19}$$

Now expanding $\mathbf{v}$ as a Taylor series about $\xi = 0$ and integrating, we obtain

$$\mathbf{U} = \mathbf{v}(0) + \frac{c^2}{6}\frac{\partial^2\mathbf{v}}{\partial\xi^2}\bigg|_{\mathrm{o}} + \cdots \tag{20}$$

and

$$\boldsymbol{\omega} = \tfrac{1}{2}\nabla\times\mathbf{v}(0) + \frac{e^2}{(2-e^2)}\mathbf{d}\times\mathbb{e}(0)\cdot\mathbf{d} + \frac{c^2}{20}\frac{\partial^2}{\partial\xi^2}\nabla\times\mathbf{v}|_{\mathrm{o}} + \frac{c^2e^2}{10(2-e^2)}\frac{\partial^2}{\partial\xi^2}\mathbf{d}\times\mathbb{e}\cdot\mathbf{d}|_{\mathrm{o}} + \cdots. \tag{21}$$

Equations [20] are series expansions in powers of $(c/l)^2$. It is difficult to imagine a flow through a confined channel for which $(c/l)^2$ could be much greater than about 10^{-2}. The particles would either break or become stuck. Thus, for simply calculating the position and orientation of a spheroid within a confined flow the zeroth-order solution of [20] and [21] is quite adequate. The zeroth-order solution is the first term on the r.h.s. of [20] and the first two terms of the r.h.s. of [21]. The zeroth-order solution could also have been derived by a truncation of the infinite series expansion of Brenner (1964, 1966). Workers at the Center for Composite Materials (Givler *et al.* 1983; Gillespie *et al.* 1985) have produced a numerical simulation of spheroid position and

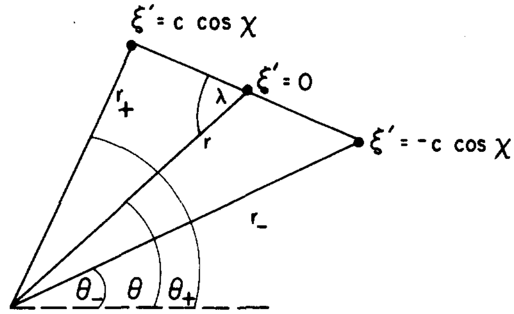

Figure 3. Trigonometry for obtaining the coordinates of the spheroid axis projected onto the plane of flow.

orientation which takes into account the flow velocity and velocity gradient only at the spheroid center. It is clear from [20] and [21] and the magnitude of $(c/l)^2$ that their method of numerical simulation is sufficiently accurate even for nonuniform velocity gradients. The zeroth-order solution predicts no motion of the center of the particle relative to the velocity field. Particle migration is a second-order effect and is important only insofar as it affects heat transfer, mixing or other characteristics besides particle position and orientation.

We will calculate U_θ to second order, but retain only the zeroth-order terms for U_r and ω. Thus, the radial particle velocity U_r is simply v_r, given in [12]. Harris & Pittman (1976) included only the zeroth-order terms in [21] and found an approximate solution for the orientation of a prolate spheroid as a function of position within the convergent channel. Their approximations are equivalent to assuming, first, that we can neglect terms involving $\partial v_r/\partial\theta$ (the "Couette shear") relative to terms involving $\partial v_r/\partial r$ (the "accelerating shear") and, second, that $c \gg b$. We have already assumed $c \gg b$ by keeping terms of order $(c/r)^2$ while dropping terms of order $(b/r)^2$. (Spheroids for which $c \gg b$ are often called "rodlike particles".) Let λ be the clockwise angle between the r-axis and the projection of the spheroid axis onto the plane of flow (see figure 3), and χ be the angle between the spheroid axis and the plane of flow. Then, with the above approximations, Harris & Pittman (1976) found that

$$\tan\lambda = \tan\lambda_i \frac{r^2}{r_i^2} \qquad [22]$$

and

$$\tan\chi = \tan\chi_i \frac{r}{r_i}, \qquad [23]$$

where the subscript "i" indicates an initial value.

We now proceed to the calculation of U_θ for a rodlike particle. Equation [18] can be rewritten as

$$2\frac{\partial}{\partial c}(c\mathbf{U}) = \mathbf{v}(\xi = c) + \mathbf{v}(\xi = -c). \qquad [24]$$

Expanding the r.h.s. of [24] to order $(c/r)^2$ and integrating with respect to c will yield an expression for U_θ correct to order $(c/r)^2$. Figure 3 shows the projection of the spheroid axis onto the plane of flow. Let the ξ'-axis be the projection of the ξ-axis (the spheroid axis) onto the plane of flow. Let the coordinates of the center of the spheroid axis ($\xi' = 0$), the positive focus of the spheroid axis ($\xi' = +c\cos\chi$) and the negative focus of the spheroid axis ($\xi' = -c\cos\chi$) be respectively, (r, θ), (r_+, θ_+) and (r_-, θ_-). Using trigonometry we obtain

$$r_+ = (c'^2 + r^2 - 2c'r\cos\lambda)^{\frac{1}{2}}, \qquad [25]$$

$$r_- = (c'^2 + r^2 + 2c'r\cos\lambda)^{\frac{1}{2}}, \qquad [26]$$

$$\theta_+ = \theta + \sin^{-1}\left(\frac{c'\sin\lambda}{r_+}\right) \qquad [27]$$

and

$$\theta_- = \theta - \sin^{-1}\left(\frac{c'\sin\lambda}{r_-}\right), \quad [28]$$

where $c' = c\cos\chi$. Projecting the fluid velocities at the foci onto the θ-direction as defined at the spheroid center, we obtain

$$2\frac{\partial}{\partial c}(cU_\theta) = v_r(\xi' = c\cos\chi)\sin(\theta_+ - \theta) - v_r(\xi' = -c\cos\chi)\sin(\theta - \theta_-). \quad [29]$$

Then inserting [12] and [25]–[28] into [29], expanding the r.h.s. and integrating, yields

$$U_\theta = -\frac{2A}{3r}\left(\frac{c}{r}\right)^2\cos^2\chi\,\{\sin^2\lambda\sin(2\theta) - \cos\lambda\sin\lambda\,[\cos(2\theta) - \cos(\alpha)]\}. \quad [30]$$

Note that [30] does not reduce to [15] in the limit $c \to 0$ because we have assumed $c \gg b$.

For convergent flow, the first term on the r.h.s. of [30] predicts migration toward the wall, while the second term predicts motion either toward or away from the wall depending upon the orientation. For rodlike particles aligned with the r-axis, $U_\theta = 0$. We wish to determine the average particle motion by averaging over all possible initial orientations in λ. Assume the orientations to be uniformly distributed in λ at $r = r_i$ and define

$$\langle f\rangle_\lambda = \frac{1}{\pi}\int_0^\pi f(\lambda_i)\,d\lambda_i. \quad [31]$$

Then combining [22] and [23] with [30] and [31] yields

$$\langle U_\theta\rangle_\lambda = \frac{-2A}{3}\left(\frac{c}{r_i}\right)^2\sin(2\theta)\cos^2\chi\frac{1}{r\left[1+\left(\frac{r}{r_i}\right)^2\right]} \quad [32]$$

Thus, in covergent flow, the average spheroid motion is toward the wall just as in the spherical case.

4. TEMPERATURE CHANGE ACROSS A CONVERGENT CHANNEL

The purpose of this section is to calculate the temperature change across a convergent channel due to particle migration and the difference in specific heat capacities between the fluid and the particles. For the convergent flow defined by [12] it is clear that the analogue of [7] in cylindrical coordinates is

$$\frac{dT}{dt} = \left(\frac{c_f - c_s}{c_e}\right)U_\theta\phi\frac{1}{r}\frac{\partial T}{\partial\theta}. \quad [33]$$

From [4] and [13] or [16] it is clear that $\phi = \text{const}$ is a solution to the equation of conservation of particles. If we regard the concentration as uniform at the entrance to the channel, then we can take ϕ to be a constant throughout the channel. The temperature gradient $(1/r)\partial T/\partial\theta$ can be determined only by solving the time-dependent energy equation including conduction and shear heating. Since we wish to study only the effects of particle migration, we take the lateral temperature gradient to be a constant G and calculate the temperature change for a range of constants.

Let the suspension flow into the channel at $r = r_i$ and out of the channel at $r = r_o < r_i$. Then the temperature change ΔT across the channel is

$$\Delta T = \left(\frac{c_f - c_s}{c_e}\right)\phi G\int_{r_i}^{r_o} U_\theta\frac{dt}{dr}\,dr. \quad [34]$$

From [12], we obtain

$$\frac{dt}{dr} = \frac{r}{A[\cos(2\theta) - \cos(\alpha)]} \quad [35]$$

Thus [34] becomes

$$\Delta T = \left(\frac{c_f - c_s}{c_e}\right)\frac{\phi G}{A}\frac{1}{\cos(2\theta)-\cos(\alpha)}\cdot\int_{r_i}^{r_o} rU_\theta \, dr. \qquad [36]$$

We first consider the temperature change due to the migration of spheres. Inserting [15] into [36] and integrating yields

$$\Delta T = \left(\frac{c_f - c_s}{c_e}\right)\frac{2\phi G R^2}{3}\frac{\sin(2\theta)}{\cos(2\theta)-\cos(\alpha)}\frac{r_i - r_o}{r_o r_i}. \qquad [37]$$

We next calculate the temperature change due to the migration of rodlike particles. We set $\chi = 0$ in [32], thus setting an upper bound on ΔT. Combining [32] with [36], we obtain

$$\Delta T = \left(\frac{c_f - c_s}{c_e}\right)\frac{2\phi G}{3}\left(\frac{c}{r_i}\right)^2\frac{\sin(2\theta)}{\cos(2\theta)-\cos(\alpha)}r_i\left[\frac{\pi}{4} - \tan^{-1}\left(\frac{r_0}{r_i}\right)\right]. \qquad [38]$$

5. INJECTION MOLDING

Injection molding is an automated process for the conversion of a quantity of solid polymer into a large number of identical objects. The polymer is melted by viscous heating by a rotating screw. The molten polymer is then forced through a convergent channel (called a gate) into a thin mold cavity where it cools. After cooling, the molded part is ejected from the mold and the cycle begins again. Often refractory spheres or rods of glass or carbon are added to the polymer in order to improve the mechanical properties of the final product. The purpose of this section is to apply [37] and [38] to the injection molding of a polymer composite. For this application, [37] and [38] are approximations since the molten polymer is non-Newtonian and particle interactions in general cannot be neglected.

One of the advantages of forcing the melt through a convergent channel is that the resulting shear heating lowers the melt viscocity, thus facilitating the flow into the mold cavity (Chung 1976). This study arose out of the concern that the cooling due to particle migration might offset the heating due to viscous dissipation. Fortunately, the temperature drop due to particle migration is negligible. Typical parameters for a polymer–glass composite are $c_f = 3 \times 10^7$ erg/g °C, $c_s = 0.8 \times 10^7$ erg/g °C, $\phi = 0.2$, $R = c = 0.1$ mm, $r_o = 0.15$ cm, $r_i = 2.5$ cm and $\alpha = 45°$. Only temperature drops of the order of 10°C are important and, from [37] and [38], they are possible only if the lateral temperature gradient is of the order 10^4–10^5 °C/cm, which is unrealistic. Arbitrarily high temperature drops are possible for streamlines close enough to the wall simply because fluid close to the wall takes a long time to travel through the channel. However, [37] and [38] are invalid close to the wall since wall effects have been neglected.

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